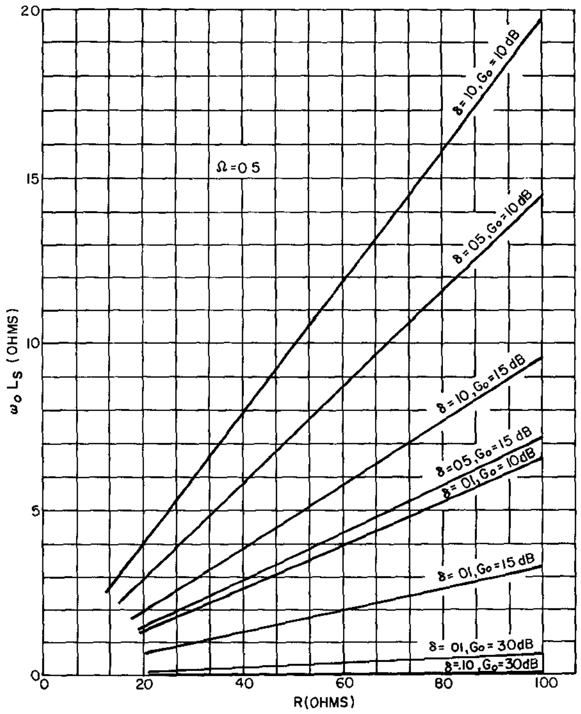

Fig. 6. Limiting series reactance for tunnel diode amplifier stability, $\Omega=0.5$.

To satisfy the Routh-Hurwitz stability criterion which requires that all roots of (2) be in the left half of the complex frequency plane p, the following conditions must be satisfied:

$$a_0 \geq 0 \quad (3)$$

$$a_1 > 0 \quad (4)$$

$$\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0. \quad (5)$$

In order to analyze the stability of a particular tunnel diode amplifier using the above criterion, only the diode parameters, the source impedance R_0, and the tuning inductance L need to be known. The required value of the resistance R_0 to realize a specified gain at the midband frequency f_0 can be evaluated using the well-known gain equation

$$G_0 = 20 \log_{10} \frac{|R'| + R_0}{|R'| - R_0} \text{ dB} \quad (6)$$

where R', the equivalent shunt negative resistance at the diode terminals, is given approximately by

$$R' = R\frac{1 - \delta(1 - \Omega^2)}{(1 - \Omega^2)},$$

where

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}},$$

$$\delta = R_S/R,$$

$$\Omega = \omega_0/\omega_c,$$

and

$$\omega_c = \frac{1}{RC}\sqrt{\frac{1-\delta}{\delta}} = \text{diode resistive cutoff frequency.}$$

The derived stability criterion can be used to determine the limiting values of diode series reactance $\omega_0 L_S$ for the stable operation of tuned amplifiers. In this case limiting values of L_S which satisfy inequalities (3) through (5) are to be determined. It was found numerically, however, that if inequality (5) is satisfied, inequalities (3) and (4) will also be satisfied over the range of interest. The numerical results for the limiting values of diode series reactance as functions of diode negative resistance are shown in Figs. 4 through 6 for a wide range of diode parameters and amplifier gain. Each figure contains curves for one value of the normalized frequency $\Omega=\omega_0/\omega_c$ and each curve is plotted for a specified gain and resistance ratio, δ.

It should be pointed out that the stability criterion derived for the tuned tunnel diode circuit is shown to be applicable to the complete circuit of an amplifier using an out-of-band stabilizing network such as that shown in Fig. 1. It is assumed that the parasitic elements are small compared with the main elements and that all tank circuits are resonant at the same frequency. These assumptions are usually valid in the case of a well-designed amplifier. In some amplifier designs, however, stabilizing networks are not used, and instead, out-of-band stability is realized by increasing the frequency selectivity of the diode circuit to be compatible with that of the circulator. The increased selectivity is achieved by adding a capacitance at the diode terminals. In this case the parasitic inductance in series with the added capacitance would have a very large effect on the amplifier out-of-band stability. In fact, this series inductance can be an order of magnitude smaller than the diode series inductance L_S and still cause instability.

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Transient Thermal Behavior of Latching Ferrite Phase Shifters

Latching ferrite phase shifters [1], [2] have been under investigation and development for a number of years as digital phase control elements in electronically steerable arrays [3]. This device employs the remanent magnetization available in a closed magnetic circuit to eliminate the large and inefficient electromagnets associated with previous ferrite phase shifters, and to provide for microsecond speed switching with low energy. The basic element of a waveguide latching phase shifter is a rectangular toroid of square hysteresis loop ferrimagnetic material with an axial magnetizing wire. Current pulses are used to switch the toroid magnetization between the two possible remanent states. The effect of changing the direction of remanent magnetization is to perturb the propagation constant of the ferrite loaded guide in the order of 10 percent, creating a differential phase shift.

From an examination of the mode of operation of this device it is seen that any effects which vary the value of remanent magnetization can change the differential phase shift. The steady-state aspects of phase variation due to RF heating and ambient temperature changes, and methods to minimize these effects have been reported by a number of investigators [4], [5]. This correspondence is concerned with the transient effects of changes in applied RF power to which this type of phase shifter could be subjected in array applications.

The geometry for thermal analysis is shown in Fig. 1. To simplify calculations we assume that RF power produces uniformly distributed heating within the body of the ferrite, that cooling occurs only by conduction in the X direction to the top and bottom waveguide walls, and that the quadrant marked A conducts one quarter of the dissipated power, ignoring the small area above and below the slot. These assumptions enable us to apply the solution of Carslaw and Jaeger [6] for transient heat flow in an internally heated infinite slab between two heat sinks.

$$\Delta T(x, t) \cong \frac{A_0 L^2}{2K} \cdot \left\{ 1 - \frac{X^2}{L^2} - \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \cdot \cos\frac{(2n+1)}{2L}\pi x \cdot \exp\left[-\alpha(2n+1)^2\pi^2 \frac{t}{4L^2}\right] \right\} \quad (1)$$

where

ΔT = temperature difference between the ferrite surface ($X=L$) and a plane in the ferrite at position X,

A_0 = heat dissipation per unit volume,

K = thermal conductivity = 0.015 cal/s (cm)^2/°C(cm) for ferrite,

α = diffusivity = $K/\rho c$ = 0.013 cm^2/s,

ρ = density = 5.6 g/cm^3 for ferrite, and

c = specific heat = 0.2 cal/g(°C) for ferrite.

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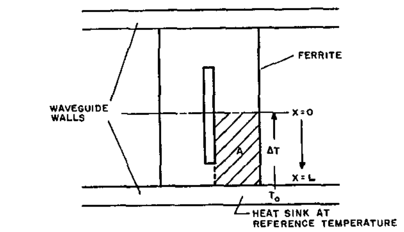

Fig. 1. Cross section of latching ferrite phase shifter for thermal analysis.

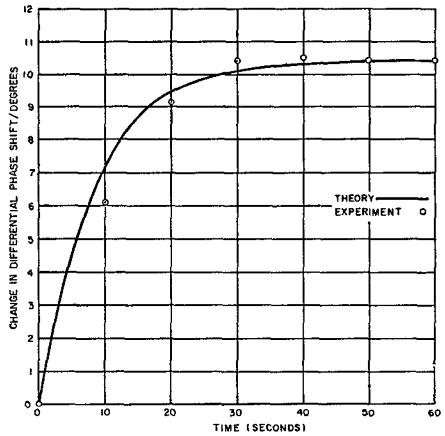

Fig. 2. Transient variation of phase due to RF heating of X-band latching ferrite phase shifter.

The highest temperature exists at the mid-plane ($x=0$)

$$\Delta T(0, t) = \frac{A_0 L^2}{2K}\left\{1 - \frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3} \cdot \exp\left[-\alpha(2n+1)^2\pi^2\frac{t}{4L^2}\right]\right\}. \quad (2)$$

The solution of interest will be for values of t sufficiently large to bring ΔT to within 10 to 100 percent of its steady-state value. For this range of t only the first term of the series is required:

$$\Delta T(0, t) \cong \frac{A_0 L^2}{2K} \cdot \left\{1 - 1.032 \exp\left[-\alpha\pi^2\frac{t}{4L^2}\right]\right\}. \quad (3)$$

This equation is a good approximation to the standard transient relationship in an electrical network with a time constant τ of $4L^2/\alpha\pi^2$. For a typical X-band waveguide phase shifter $L=0.2$ inch and $\tau=8.05$ seconds. For small changes in temperature the change in remanent magnetization and differential phase shift can be expected to be approximately linearly proportional to ΔT. The validity of this computation was evaluated experimentally with an X-band latching phase shifter operating at 200 watts average power. With the device latched in one state the power was suddenly applied and phase length recorded as a function of time. After removing the power and allowing sufficient time for cooling to the original equilibrium temperature, the latched state was reversed and the experiment repeated. The differential phase shift between the states was then calculated as a function of time. A slow residual change of the phase in the order of 1 degree per 15 seconds was found to exist at the end of the ferrite heating transient period due to slow heating of waveguide components. This drift component was removed from the experimental data and the resultant data plotted in Fig. 2. Equation 2 was normalized to the experimental data at the equilibrium point reached in 60 seconds and is shown as the theoretical curve in the figure.

From this data and calculations it is clear that latching ferrite phase shifters respond slowly to changes in applied power. Moreover, when using these devices with pulsed RF power, no significant change in temperature and resulting phase can take place during a single pulse of duration less than 10 milliseconds.

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A High-Precision Wideband Wavemeter for Millimeter Waves

The purpose of this correspondence is to describe a high-precision wideband wavemeter for millimeter waves. At longer wavelengths usually tunable cylindrical cavities are used, which are half a guide wavelength long [1], [2]. Going down to millimeter waves two disadvantages arise. One is the low Q factor as the result of increasing wall losses and the other is the low accuracy of measurements as the result of the smaller dimensions of the cavity. Both disadvantages can be overcome by a cavity of larger volume. The Q factor increases with the volume, and the accuracy is proportional to the length of the cavity. That means that the cavity is several or many wavelengths long and a mode of higher axial order ($H_{01n}{}^{\circ}$ with $n>1$) must be used. But then the problem arises that n has to be known for the determination of frequency. This can be achieved using two cavities of equal diameter but different lengths l_1 and l_2 which are independently coupled to the same waveguide.

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From waveguide theory it can be shown that resonances in both cavities at the same frequency can occur only if $n_1/n_2=l_1/l_2$, where n_1, n_2 are the numbers of half wavelengths in the two cavities, respectively. If n_1 and n_2 are chosen such that they have no common divider there is no other coincidence of resonance at the ratio l_1/l_2 except at about the double, threefold, etc. frequency.

The wavemeter can now be designed as follows. Two circular cavities of equal diameter are coupled to the same waveguide. The $H_{01n_1}{}^{\circ}$ and $H_{01n_2}{}^{\circ}$ modes, respectively, are excited. The two pistons which change the length of the cavities can be moved by a spindle drive of equal pitch. The rotation of the spindle is transferred to mechanical counters by toothed-wheel gears. The gear ratios are different for each resonator; they differ by the factor n_1/n_2 so that the two counter readings are equal if the ratio l_1/l_2 of the cavity lengths is exactly equal to n_1/n_2. If both resonators show resonance at the same frequency and if at the same time the counter-readings are equal, the calibration is valid. The reading itself is connected to the frequency by a conversion table. In this way the frequency can easily and quickly be determined.

A wavemeter for the 2 mm band has been built, covering the frequency range from 110 to 150 GHz. The number of half wavelengths in the two cavities was chosen $n_1=45$ and $n_2=31$, respectively. The cavities are sections of a circular helix waveguide similar to that described by Rose [3]. An insulated copper wire of 0.1 mm diameter embedded in a mixture of epoxy resins and tin oxide builds up the helix with an inner diameter of 5.7 mm. Since in a helix the circumferential currents are preferred, the $H_{0m}{}^{\circ}$ modes will be excited. Other modes with longitudinal currents will be highly suppressed [4].

With the chosen diameter a high Q factor is obtained for the $H_{01}{}^{\circ}$ mode whereas the $H_{03}{}^{\circ}$ mode is under cutoff. Both cavities are reaction type coupled through holes in the small side of an RG 136/U waveguide. The position of the holes in the cavity end plates is such that only the $H_{01}{}^{\circ}$ mode will be excited [5]. The tuning is provided by noncontacting copper pistons. The loaded Q factor is about 10^4. Figure 1 shows a schematic drawing and Fig. 2 a photo of the wavemeter.

The calibration has been performed by means of gas resonance absorption lines, utilizing molecular frequency standards, described by Schulten [6]. One absorption line of the CO molecule and four of the OCS molecule lie in the frequency range between 109 and 146 GHz. They are known (with a relative error of 10^{-6}) from spectroscopic measurements. These five calibration points can be used to calibrate the whole range with the aid of waveguide theory. For every integer of the counter reading the related frequency has been calculated by a computer. From these calculations [7] it turns out that the relative error was smaller than 10^{-4} over the whole range from 109 to 150 GHz. Higher accuracy can only be obtained if temperature and humidity are considered. It seems possible to build a wavemeter of this kind with the same accuracy down to a wavelength of one or even 0.5 mm.